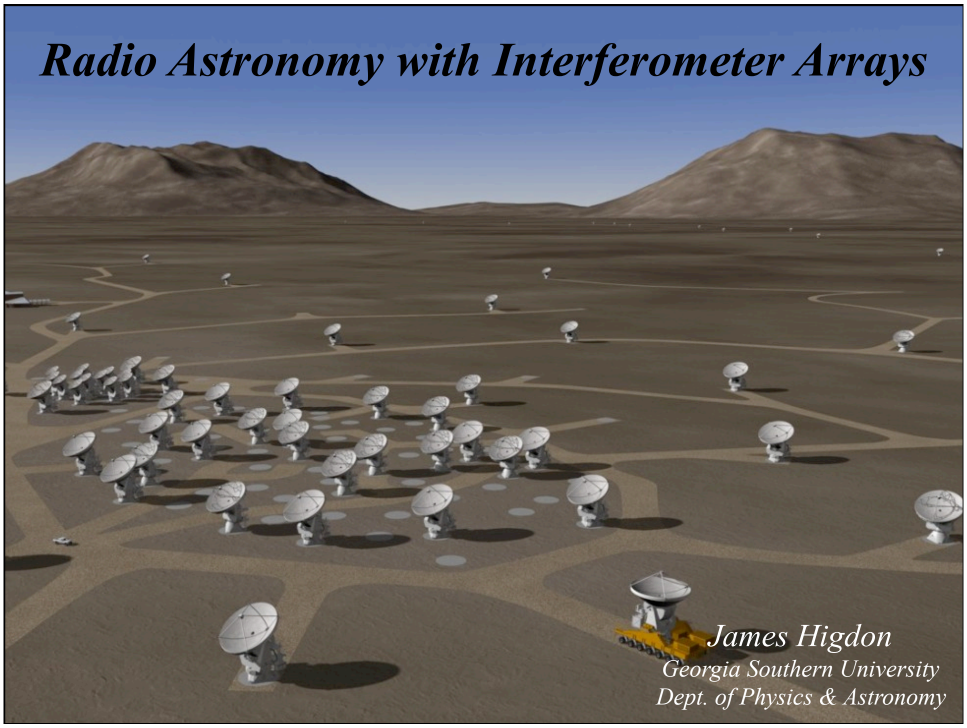
Radio Astronomy with Interferometer Arrays



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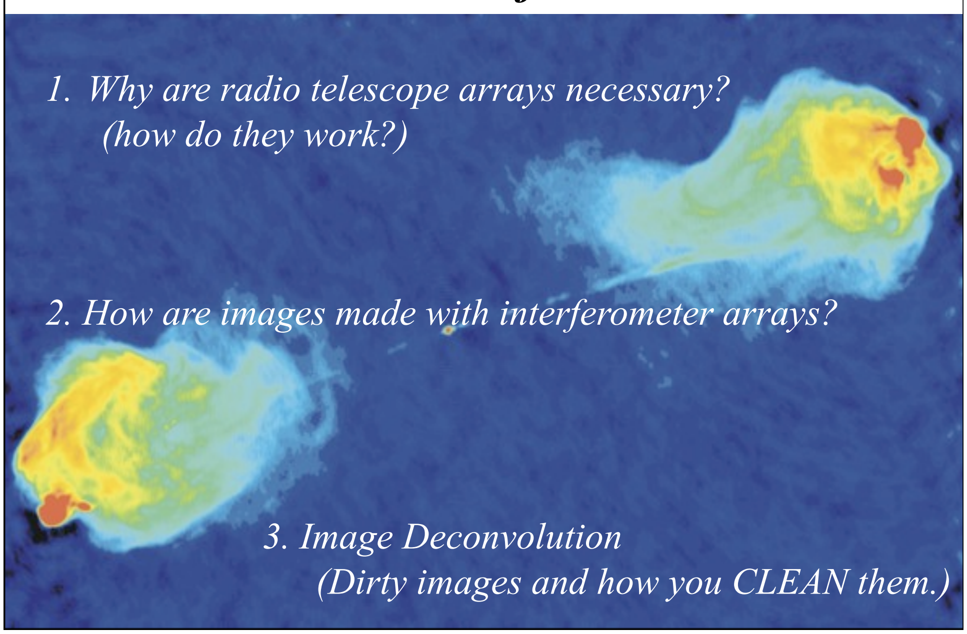
Georgia Southern University
Dept. of Physics & Astronomy

Outline of Talk



1. *Why are radio telescope arrays necessary?*
   (how do they work?)
2. *How are images made with interferometer arrays?*
3. *Image Deconvolution*
   (Dirty images and how you CLEAN them.)

Telescopes

Since we can't go there, we use telescopes to collect (faint) EM-radiation they emit.

"OK … so Venus has phases … but check-out Romeo & Juliet over there … wooo!"

Astronomers use telescopes to determine properties of a source's radiant energy:

* *intensity (Luminosity)*
* *frequency distribution*
* *polarization state*
* *coherence (masers)*
* *angular distribution on the sky*

→ ***Astronomers are never satisfied!***

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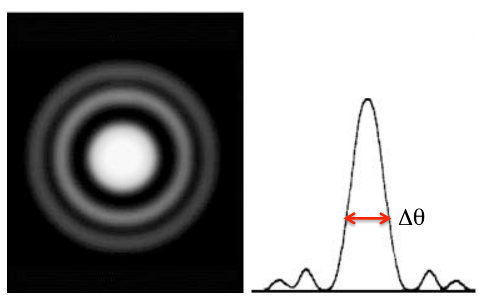
Angular Resolution

Depends simply on the mirror's diameter and wavelength of photons being observed.

$$\Delta\theta = \frac{1.22}{(D/\lambda)}$$

Increasing the diameter of the mirror increases the angular resolution, i.e., you can resolve finer and finer detail in an image.



Can be understood in terms of wave interference.

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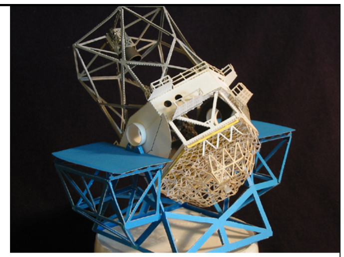
Angular Resolution

$$\Delta\theta(arc\sec) = \frac{0.00025}{(D_m / \lambda_{nm})}$$



Keck 10m optical telescope *(λ = 500 nm)*:
(D/λ = 20-million)

$\Delta\theta$ = 0.00025 x 500 / 10 = ***0.013 arcsec***

1" = 1/3,600th Degree
(full-moon spans 1800")

Keck 10m optical telescope *(λ = 20 cm)*:
(D/λ = 50)

$\Delta\theta$ = 0.00025 x 2x10^8 / 10 ***= 5,000 arcsec***
= 1.4 degrees
~3 full-moons!!!

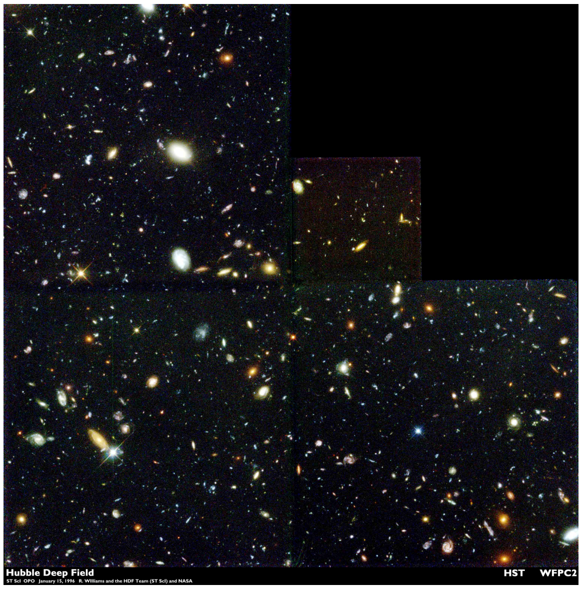
5

Arecibo Optimus Maximus

At 20cm: $\Delta\theta$ = 167 arcsec

6

You may crave higher angular resolution



The Hubble Deep Field is 2.5 arcminutes in size. In it are about 3,000 galaxies, many of which are extremely distant and young.

Some of these possess AGN powered by accretion onto still growing super-massive blackholes.

Many of the most distant galaxies appear to be merging implying galaxy "assembly" in the early Universe.

We want to study these at radio wavelengths. Uh-oh…

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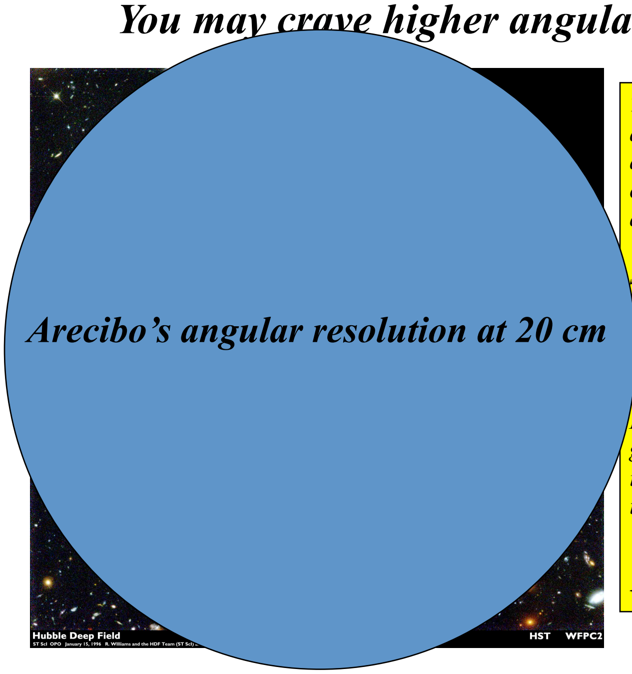
You may crave higher angular resolution



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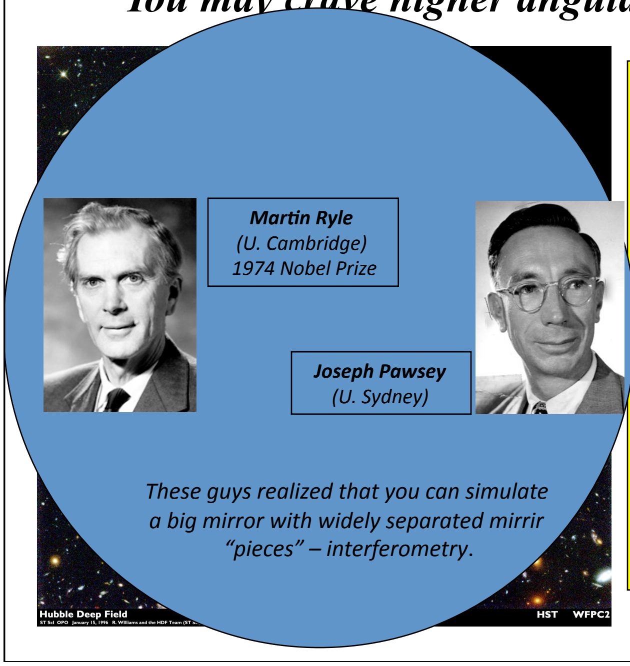

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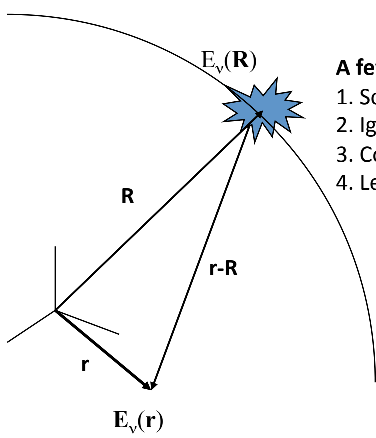
1. Radiant Energy from Astronomical Objects

For distant distribution of currents at **R**, we measure fields that propagate to **r**. What form does $\mathbf{E}_\nu(\mathbf{r})$ take?



A few simplifying assumptions:

1. Source is extremely distant (depth information lost).
2. Ignore polarization for now.
3. Consider single frequency components: $\mathbf{E}_\nu(\mathbf{R})$, $\mathbf{E}_\nu(\mathbf{r})$
4. Let space to be "empty".

We have a simple form for the **E**-fields measured at **r** from source fields at **R**:

$$\mathbf{E}_\nu(\mathbf{r}) = \int \mathrm{E}_\nu(\mathbf{R}) \frac{e^{2\pi i \nu |\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|} dS$$

(i.e., Spherical waves)

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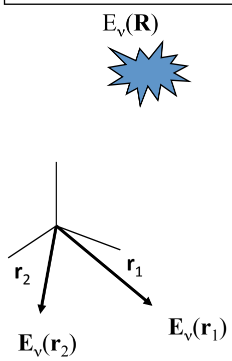
The Spatial Coherence Function: $V_\nu(\mathbf{r}_1,\mathbf{r}_2)$

One way to characterize the Electric fields is to determine how the fields measured at $\mathbf{r}_1$ compare to those measured at $\mathbf{r}_2$, i.e, its spatial correlation.



Spatial Coherence Function:

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \langle \mathbf{E}_\nu(\mathbf{r}_1)\mathbf{E}^*_\nu(\mathbf{r}_2)\rangle$$

… using result from last slide gives…

$$V_\nu(\mathbf{r}_1,\mathbf{r}_2) = \langle \iint E_\nu(\mathbf{R}_1)E^*_\nu(\mathbf{R}_2)\frac{e^{-2\pi i\nu|\mathbf{R}_1-\mathbf{r}_1|/c}}{|\mathbf{R}_1-\mathbf{r}_1|}\frac{e^{2\pi i\nu|\mathbf{R}_2-\mathbf{r}_2|/c}}{|\mathbf{R}_2-\mathbf{r}_2|}dS_1 dS_2\rangle$$

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Basic Equation of Aperture Synthesis

$$V_\nu(u,v) = \iint I_\nu(l,m)\, e^{-2\pi i\nu(ul+vm)/c}\, dl\, dm$$

This is a Fourier integral equation: we can **invert** *it.*

$$I_\nu(l,m) = \iint V_\nu(u,v)\, e^{2\pi i\nu(ul+vm)/c}\, du\, dv$$

We can determine the intensity distribution on the sky by measuring the spatial coherence function on the "ground".

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Basic Equation of Aperture Synthesis

$$I_\nu(l,m) = \iint V_\nu(u,v)\, e^{2\pi i \nu (ul + vm)/c}\, du\, dv$$

Things to note:
1) measurements of V_ν directly related to intensity distribution on sky.
2) equation depends only on ***separations***, not absolute locations.

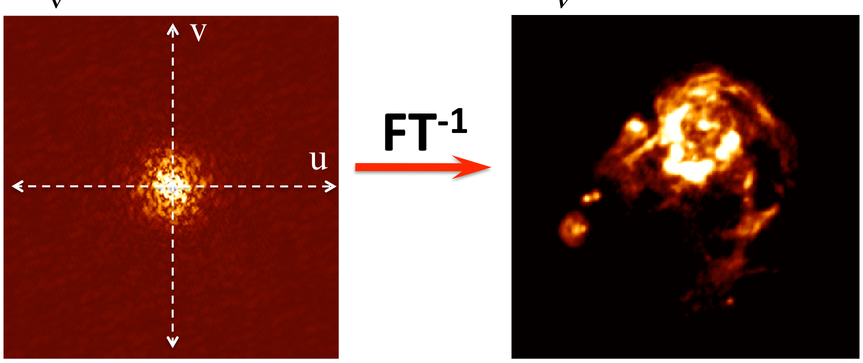
An *interferometer array* is a machine to measure V_ν's

- each telescope pair gives one V_ν
- N-telescopes → N(N-1)/2 pairs (VLA has 27-antennas: 351-pairs)

2. Imaging with Synthesis Arrays: Ideal Case

Measure spatial coherence function ***everywhere*** *then inverse Fourier-transform.*

V_ν^{True} I_ν^{True}



$$I_\nu(l,m) = \iint V_\nu(u,v)\, e^{2\pi i \nu (ul + vm)/c}\, du\, dv$$

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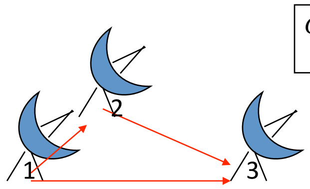
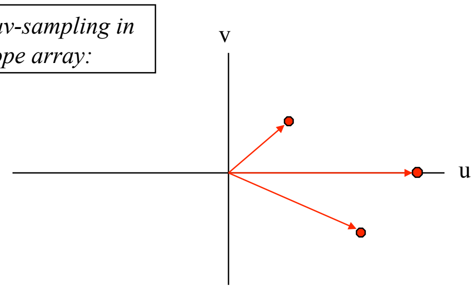
Sampling of the uv-Plane

In reality you measure visibilities only at a finite set of points in the uv-plane.

Consider the uv-sampling in a 3-telescope array:





Any real array will only ***sample*** *the* uv-plane. *You do not measure visibilities* ***everywhere***. *Can write "sampling function":* $S(\mathrm{u},\mathrm{v}) = \sum_{k=1}^{M} \delta(\mathrm{u}-\mathrm{u}_k, \mathrm{v}-\mathrm{v}_k)$

$$I_\nu(l,m) = \iint S(\mathrm{u},\mathrm{v})\, V_\nu(\mathrm{u},\mathrm{v})\; e^{2\pi i \nu(\mathrm{u}l + \mathrm{v}m)/c}\, du\, dv$$

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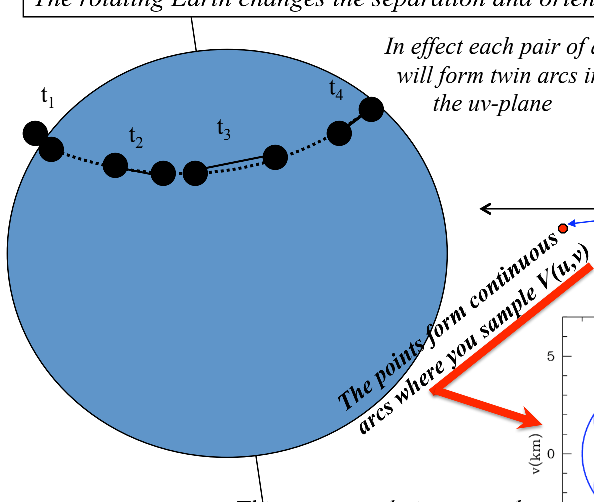
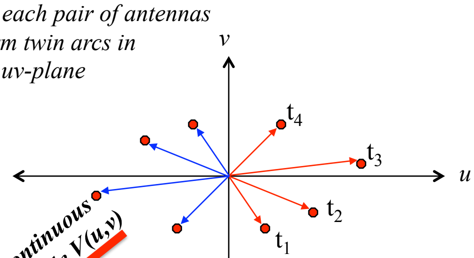
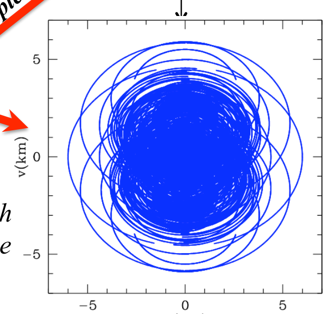
Earth Rotation Aperture Synthesis

The rotating Earth changes the separation and orientation of every antenna pair.



In effect each pair of antennas will form twin arcs in the uv-plane





This can greatly improve th "uv-coverage" and image quality.

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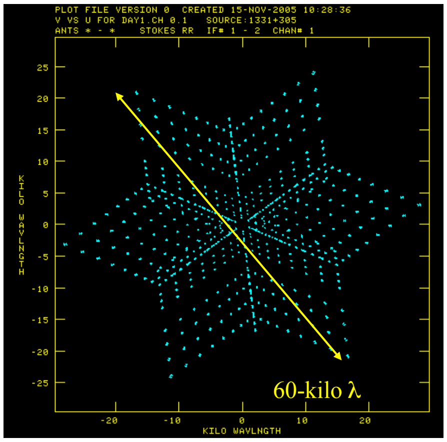
Imaging with a Synthesis Array

Ignorance/limitations in uv-plane affect the images in 3-basic ways.



1. Largest extent in uv-plane sets $\Delta\theta$.
2. Gaps in uv-plane shows up in the images as artefacts.
3. "Hole" in center limits sensitivity to large scale structure.

$$\Delta\theta(arc\,\mathrm{sec}) = \frac{2.5x10^5}{(D_m/\lambda_m)} = \frac{2.5x10^5}{6.0x10^4} = 4.2''$$

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Imaging with Synthesis Arrays: in Practice

Measure ***FINITE*** *number of* V_ν *and inverse Fourier-transform.*

(low S/N; system drifts & phase errors)

→ $\{ V_\nu(u,v) \}$

↓ ***Calibrate the Visibilities***

$\mathbf{FT}^{-1}\{ V_\nu(u,v) \}$

Not this image … but…

$$I_\nu(l,m) = \iint S(u,v)\, V_\nu(u,v)\, e^{2\pi i\nu(ul + vm)/c}\, du\, dv$$

Imaging with Synthesis Arrays: in Practice

*Measure **FINITE** number of V_ν and inverse Fourier-transform.*

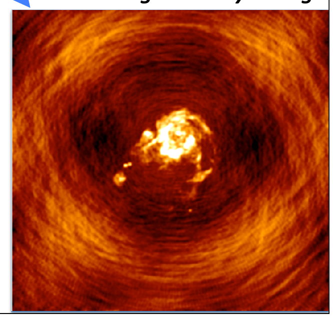
(low S/N; system drifts & phase errors)

$\{ V_\nu(u,v) \}$

Calibrate the Visibilities

$\mathbf{FT}^{-1}\{ V_\nu(u,v) \}$

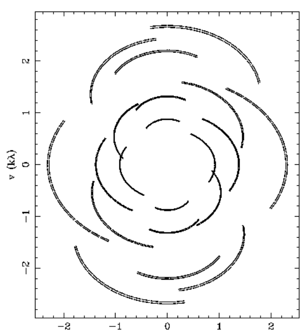
… THIS image… "Dirty" image

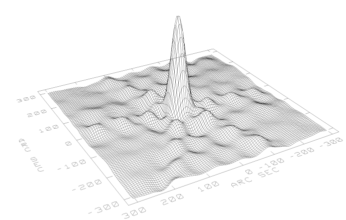


$$I_\nu(l,m) = \iint S(u,v)\, V_\nu(u,v)\, e^{2\pi i \nu (ul + vm)/c}\, du\, dv$$

"Dirty" Images in Radio Astronomy

An interferometer array gives you an image of the "true" intensity distribution of your source convolved with the "Dirty Beam".





You only measure visibilities at discrete points:

$$V(u_k, v_k) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k) V(u,v)_{\text{"True"}}$$

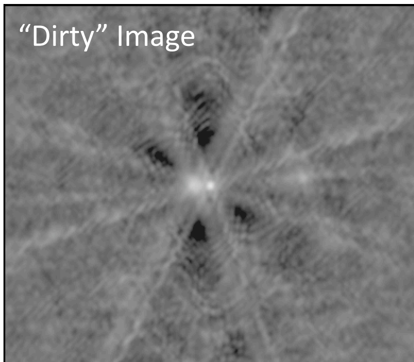
Your image reflects this "ignorance":

$$I^D(l,m) = \mathbf{FT}^{-1}\left[\sum_{k=1}^{M} \delta(u - u_k, v - v_k)\; V(u,v)_{\text{"True"}} \right]$$

$$= \mathbf{FT}^{-1}[S] * \mathbf{FT}^{-1}[V_{\text{"True"}}]$$

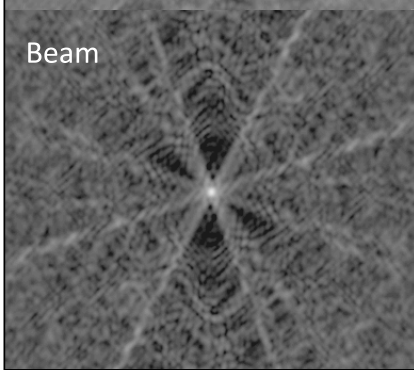
$$I^D(l,m) = B * I_{\text{"True"}}$$

"*B*" referred to as the ***Beam***
*I^D is the **Dirty Image***

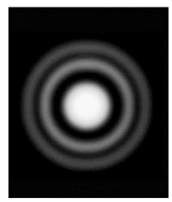


“Dirty” Image & Beam

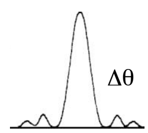
(left) VLA (λ = 6 cm) “Dirty” Image of 3C459
- *note the “side-lobes” , the dark troughs of **negative** flux and the quasi-sine/cosine noise in the background “sky”.*
- *side-lobes extend to infinity!*
- *other radio sources outside the image can contribute their own side-lobes inside image.*



*(left) VLA “Beam”, i.e., **FT**$^{-1}$\{S(u,v)\}.*
- *compare “Beam” & “Dirty” image*

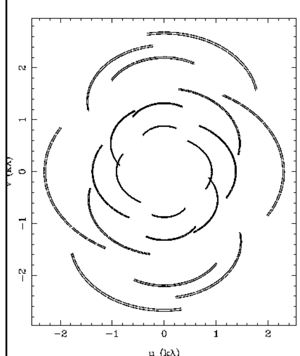


You wish!



Deconvolution

Because of S(u,v) the “true” image is convolved with the Beam: $I^D = I^{\text{“True”}} * B$.
We want algorithms to “de-convolve” the “dirty” image I^D.

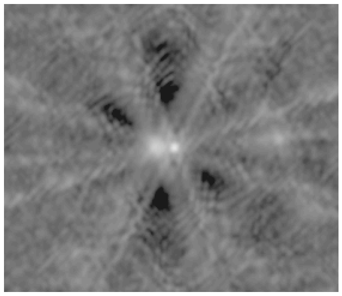


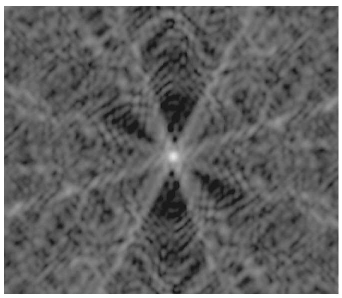
Worse:
We have no knowledge of V(u,v) beyond u_{max} & v_{max}.
*We **may** have limited knowledge of V(u,v) inside “hole”.*

We do have some physical constraints on our Ignorance:

1) *Source intensity is always positive.*
   (with a few notable exceptions)
2) *The sky “background” is mostly blank; it is not filled with sine/cosine rumble but is smooth.*
3) *There are no side-lobes extending to infinity.*
4) *Sources can be thought of as collection of point & diffuse emission.*

The CLEAN Algorithm (Hogbom 1974)





Assume that sky is mostly blank and that source is a bunch of point-sources.

1. Find pixel with peak I_ν in "Dirty" image.

2. Subtract a scaled version of the Beam from the "Dirty" image at this position, i.e.,

$$I^D_{j+1} = I^D_j - gB*\max\{I^D_j\}$$

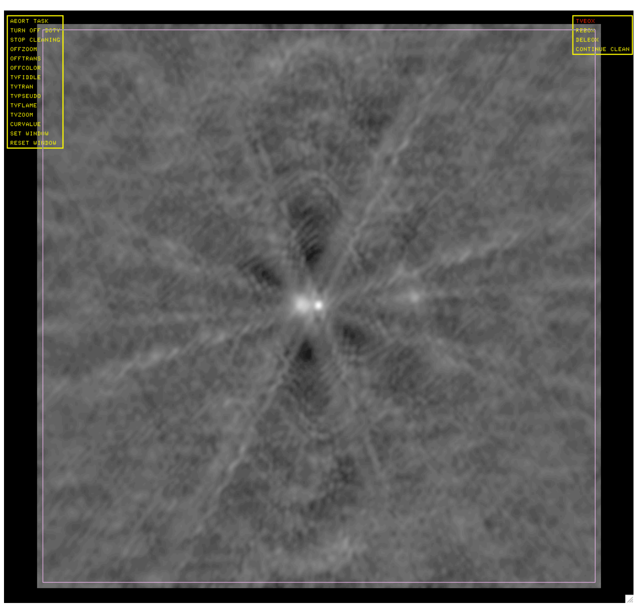
3. *In new map (I^C_j) put a delta-function at the same position with this intensity, i.e.,* $+g\delta*\max\{I^D_j\}$, and go to (1).

4. When the remainder looks "noise-like", stop and add remainder to I^c_j and convolve all the delta-functions with a nice Gaussian.

I^C_j is called the CLEAN map.

Example: CLEANing 3C459

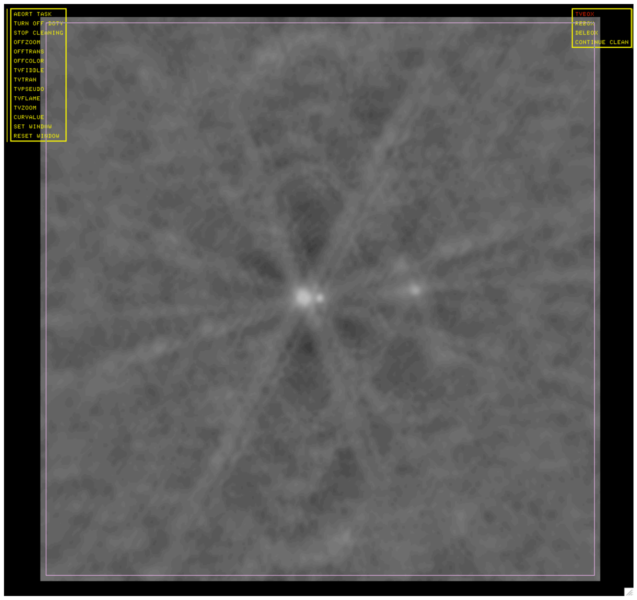
In this example a modified Hogbom CLEAN algorithm is used with "g"=0.1.



Iteration: 0
("Dirty" Image)

Example: CLEANing 3C459

In this example a modified Hogbom CLEAN algorithm is used with “g”=0.1.

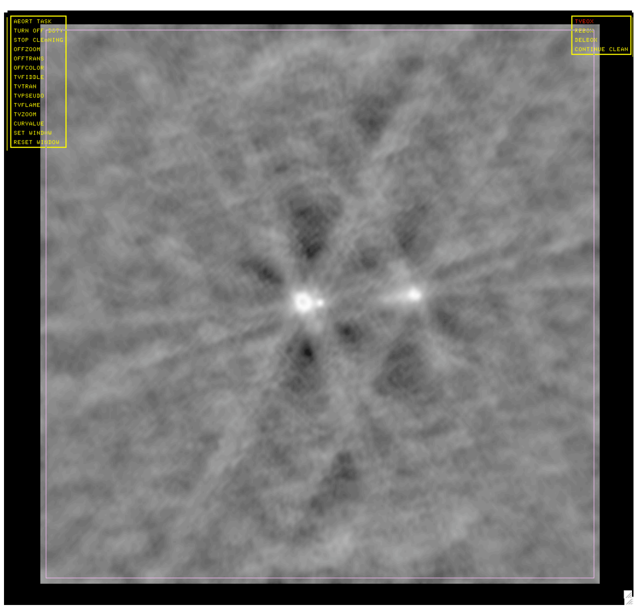


Iteration: 0
(“Dirty” Image)

Iteration: 1
(12 scaled peaks subtract)

Example: CLEANing 3C459

In this example a modified Hogbom CLEAN algorithm is used with “g”=0.1.



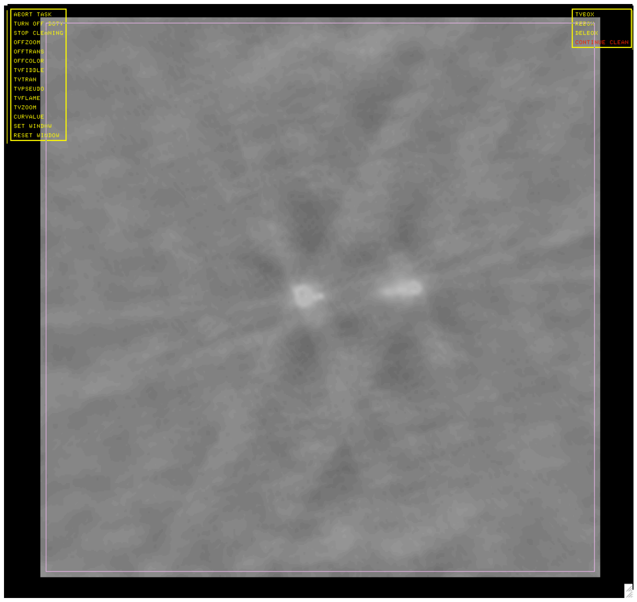
Iteration: 0
(“Dirty” Image)

Iteration: 1
(12 scaled peaks subtract)

Iteration: 2
(55 scaled peaks sub.)

Example: CLEANing 3C459

In this example a modified Hogbom CLEAN algorithm is used with "g"=0.1.



Iteration: 0
("Dirty" Image)

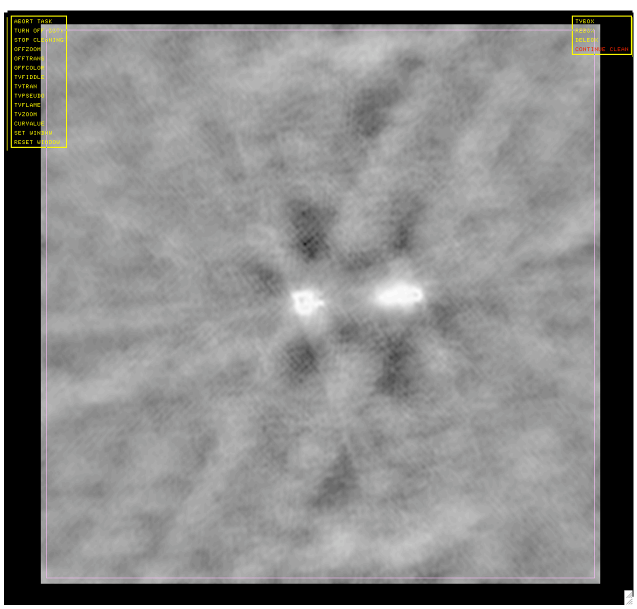
Iteration: 1
(12 scaled peaks subtract)

Iteration: 2
(55 scaled peaks sub.)

Iteration: 3
(127 scaled peaks sub.)

Example: CLEANing 3C459

In this example a modified Hogbom CLEAN algorithm is used with "g"=0.1.



Iteration: 0
("Dirty" Image)

Iteration: 1
(12 scaled peaks subtract)

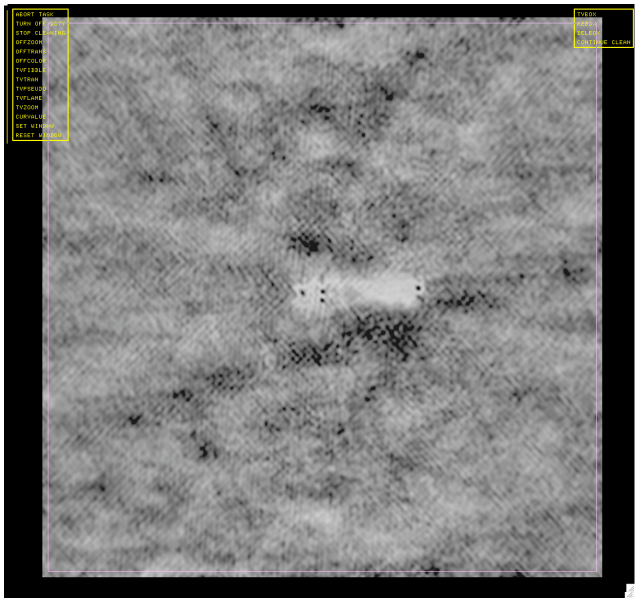
Iteration: 2
(55 scaled peaks sub.)

Iteration: 3
(127 scaled peaks sub.)

Iteration: 4
(248 scaled peaks sub.)

Example: CLEANing 3C459

In this example a modified Hogbom CLEAN algorithm is used with “g”=0.1.



Iteration: 0
(“Dirty” Image)

Iteration: 1
(12 scaled peaks subtract)

Iteration: 2
(55 scaled peaks sub.)

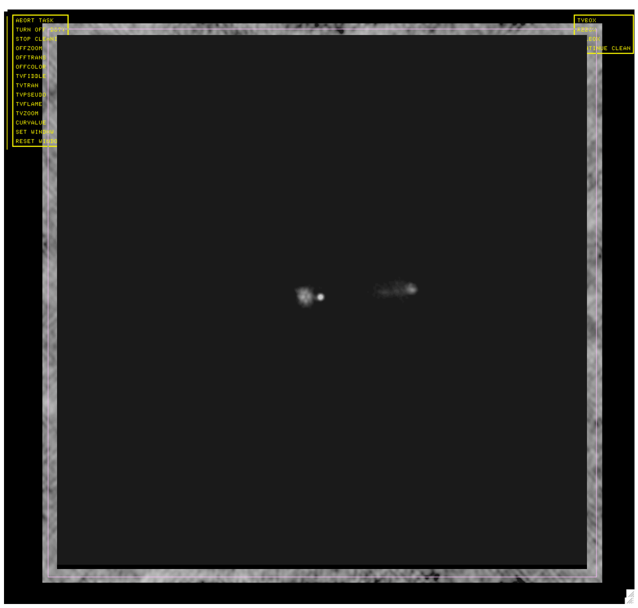
Iteration: 3
(127 scaled peaks sub.)

Iteration: 4
(248 scaled peaks sub.)

Iteration: 5
(704 scaled peaks sub.)

Example: CLEANing 3C459

In this example a modified Hogbom CLEAN algorithm is used with “g”=0.1.



Iteration: 0
(“Dirty” Image)

Iteration: 1
(12 scaled peaks subtract)

Iteration: 2
(55 scaled peaks sub.)

Iteration: 3
(127 scaled peaks sub.)

Iteration: 4
(248 scaled peaks sub.)

Iteration: 5
(704 scaled peaks sub.)

Iteration: 6
(980 scaled peaks sub.)
-hit criterion for background noise. Stop.
-convolve $\delta(l,m)$'s with 2D-Gaussian & END.

Maximum "Entropy" Deconvolution

Deconvolution can be thought of as a way of selecting the best solution (I^C) from the infinite answers allowed by our finite sampling of the (u,v) plane.

One can add the additional constraints that the "true" image must be smooth & positive.

One way to enforce this is to find solutions that maximize the image "Entropy",

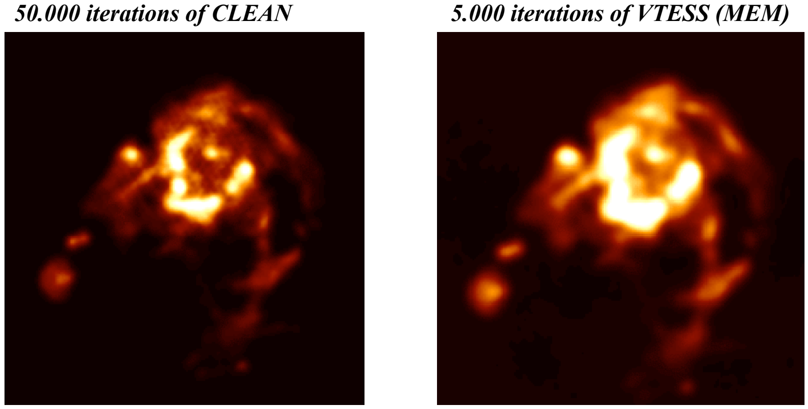
$$H = -\sum_k I_k \ln\left[\frac{I_k}{m_k}\right]$$

where the sum is over image pixels, and m_k is a "prior" image, either a blank image (or better) a low resolution image from another data set.

Maximum "Entropy" methods work better than CLEAN for extended emission.
CLEAN algorithms work better than MEM for point-sources.

CLEAN vs. MEM

50.000 iterations of CLEAN — *5.000 iterations of VTESS (MEM)*

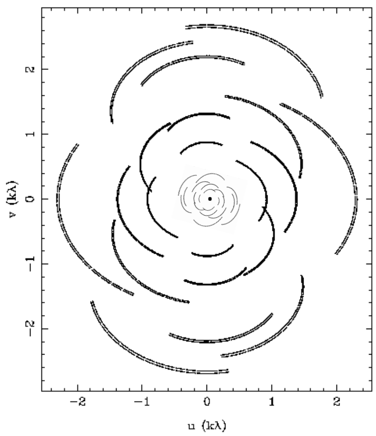


Both CLEAN and MEM can give "pleasing" deconvolved images with nearly identical fluxes. MEM is considerably faster for big (N>10^6 pixels) images.

New CLEAN variants (e.g., Adaptive Scale Pixel Bhatnagar & Cornwell 2004) are claimed to be superior to both (I havn't tried these new fangled things).

Other ways to improve your images…

Ignorance/limitations in uv-plane affect the images in 3-basic ways.



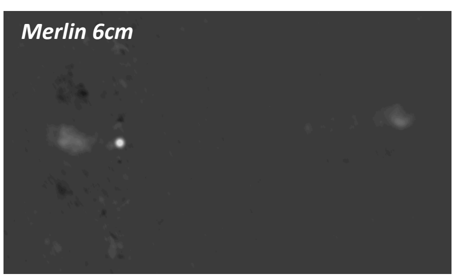
1. Largest extent in uv-plane sets Δθ.
2. Gaps in uv-plane shows up in the images ("dirty beam") as side-lobes.
3. **"Hole" in center limits sensitivity to large scale structure.**

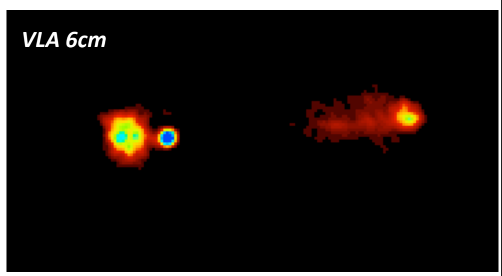
You can combine visibilities obtained with a smaller telescope array (or even at longer λ) to "fill in" the central hole.

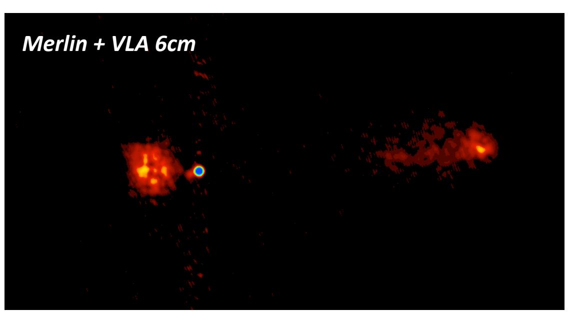
If you know the total flux from your source (from a small radio telescope) you can insert that at the very center (0-point flux) too. 33

Combining Different Arrays

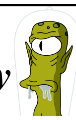
A very simple & efficient way to improve your images.








Summary




- *High angular resolution is a problem with radio telescopes due to large λ.*
- *Interferometric arrays are machines that measure the spatial coherence function of the* ***E****-fields from sources in the sky – Visibilities:* $\mathbf{V_\nu(u,v)}$
- Angular resolution in an array set by largest antenna separation.
- *Visibilities and image of source on the sky are related by a Fourier integral:*

$$I_\nu(l,m) = \iint S(u,v)\, V_\nu(u,v)\, e^{2\pi i \nu (ul + vm)/c}\, du\, dv$$

- *We measure a finite number of* $\mathbf{V_\nu(u,v)}$*. This ignorance of the "True" visibilities shows up as artefacts in the "Dirty" images.*

$$I^D(l,m) = B * I_{"True"}$$

- *CLEAN and Maximum "Entopy" algorithms exist that usually work well in finding I that best approximates* $I_{"True"}$.
- *Adding data to "fill the hole" in the uv-plane also improves the final result.*

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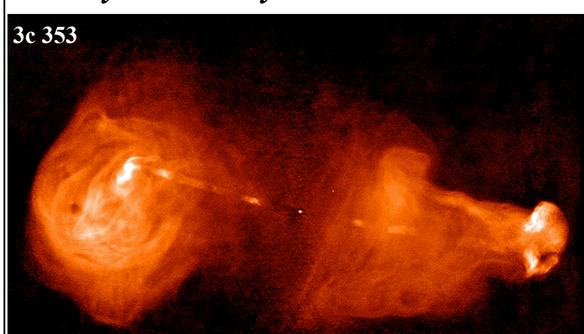
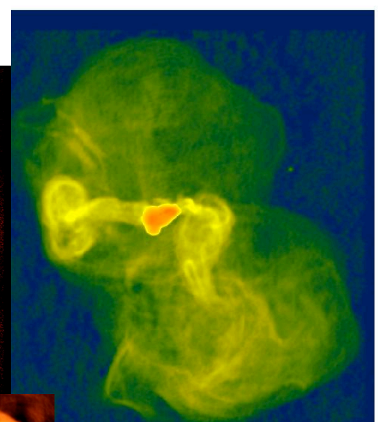
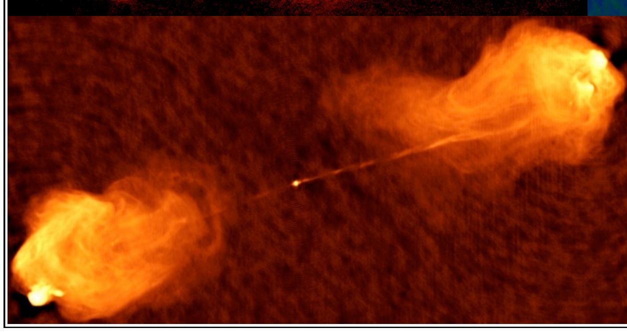
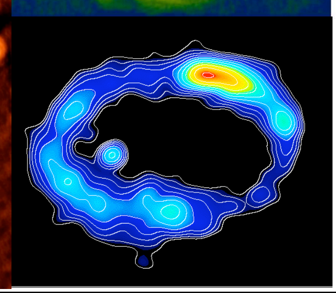
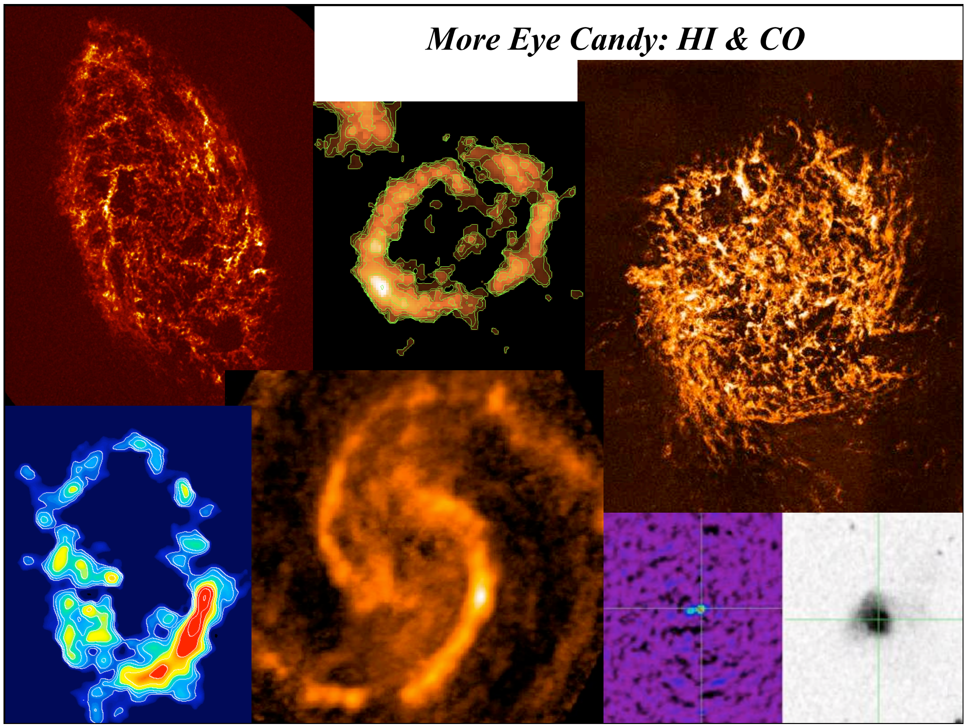
Eye Candy: Continuum










More Eye Candy: HI & CO