Let the area $A$ represent the collecting area of a telescope. The radiant energy impinging along the direction $n$ within the solid angle $d\Omega$ on the area $A$, in the time $\Delta t$ over $d\nu$ is

$$E = \Delta t \int \int I_v(\theta, \varphi) A\vec{k} \cdot \vec{n} d\Omega d\nu$$

Define $A(\theta, \varphi) \equiv \vec{k} \cdot \vec{n} A$

and separate $A(\theta, \varphi) = A_e P_n(\theta, \varphi)$

$A_e$ is referred to as the effective area in the direction $(\theta, \varphi) = (0,0)$

$$A_e = \frac{\text{Power per unit frequency recorded at antenna terminal [erg s}^{-1} \text{ Hz}^{-1}]}{\text{Flux of incident in direction (0,0) [erg s}^{-1} \text{ Hz}^{-1} \text{ cm}^{-2}]}$$

$P_n(\theta, \varphi)$ is the normalized power pattern, with $P_n(0,0)=1$, then

$$E = (1/2) A_e \Delta t \int \int I_v(\theta, \varphi) P_n(\theta, \varphi) d\Omega d\nu$$

Where the factor $1/2$ derives from the fact that a focal dipole collects only one polarization component of unpolarized radiation. $I_v$ is the specific intensity.

If $A_p$ is the physical aperture of the collecting area, then

$$\mathcal{E}_a \equiv \frac{A_e}{A_p}$$

is the aperture efficiency.
Fig. 3-2. Relation of antenna pattern to celestial sphere with associated coordinates.
Radio 3: Flux density

We define **flux density** as:

$$ S_v = \int I_v d\Omega $$

In c.g.s units its dimensions are $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$

Radio astronomers use the **Jansky** $= 10^{-26} W \text{m}^{-2} \text{Hz}^{-1}$ (prev. known as “flux unit”)

In spectroscopy, it is common to integrate the flux density across the spectral line and use the **flux integral**: $S = \int S_v d\nu$ usually expressed in $\text{Jy km s}^{-1}$

The **observed flux density**, with an antenna of **Power Pattern** $P_n$ is

$$ S_{v,obs} = \int I_v(\theta, \varphi) P_n(\theta, \varphi) d\Omega $$

and $S_{v,obs} \leq S_v$

If the solid angle subtended by the source, $\Omega_s << \text{main beam of the antenna}$, then $P_n(\theta, \varphi) \approx 1$ over $\Omega_\sigma$, and $S_{v,obs} \approx S_v$
In general, it will be desirable to map the distribution $I_v(\theta, \varphi)$ Its structural details will however be smoothed by the antenna beam. In one-D, the observed flux density with the antenna pointing in the direction $\varphi_o$ is:

$$S_{\nu, obs}(\varphi_o) = \int I_v(\varphi_o - \varphi)P_n(\varphi)d\varphi \equiv I_v * P_n$$

By the Convolution Theorem,

$$F.T.(S_{obs}) = F.T.(I) \times F.T.(P_n)$$

By dividing $F.T.(I) = F.T.(S_{obs}) \div F.T.(P_n)$ and FT back, we can recover $I(\varphi)$, but we can only do so for the harmonics for which $F.T(P_n) \neq 0$

The fine spatial structure in the source is irremediably lost.
Define:

**Beam Solid Angle**

\[
\Omega_A \equiv \int_{4\pi} P_n(\theta, \phi) d\Omega
\]

**Main Beam Solid Angle**

\[
\Omega_M \equiv \int_{\text{mainlobe}} P_n(\theta, \phi) d\Omega
\]

**Beam Efficiency**

\[
\varepsilon_b \equiv \frac{\Omega_M}{\Omega_A}
\]

Suppose the distribution \( I_v(\theta, \phi) \) is uniform throughout the sky. Then

- \( \varepsilon_b \) is the fraction of the detected power originating within
- \( 1 - \varepsilon_b \) is the fraction of the detected power arriving from everywhere else in the sky and ground.
The total power per unit bandwidth detected by the antenna while pointing in the direction \((\theta_o, \varphi_o)\) is

\[
p = A_e \int I_v(\theta, \varphi) P_n(\theta - \theta_o, \varphi - \varphi_o) d\Omega
\]

If we equate \(p\) with the thermal noise power per unit frequency interval available from a resistor at the temperature \(T_A\) which by Nyquist formula is \(p = kT_A\) then

\[
T_A(\theta_o, \varphi_o) = \left(\frac{A_e}{k}\right) \int I_v(\theta, \varphi) P_n(\theta - \theta_o, \varphi - \varphi_o) d\Omega
\]

Is the \textit{antenna temperature}

The unit of antenna temperature, 1 K, is equivalent to \(1.38 \times 10^{-23}\) W Hz^{-1}. Antenna temperature is an indication of \textit{power level}; it needs not have any relation to the temperature of any telescope component.

We also refer to the sky distribution of brightness via a \textit{brightness temperature}, with \(\lambda\) being the wavelength of observation

\[
T_B = \frac{\lambda^2 I}{2k}
\]

\(\Rightarrow\) The antenna temp. equals the all-sky integral of \(T_B\), weighted by the effective area expressed in square of \(\lambda\)
- The relationship between observed flux density and antenna temperature is then

\[ S_{obs} = \frac{2k}{A_e} T_A \]

Suppose you embed the antenna within a black box at temperature \( T \). Then \( T_B = T_A = T \)

and

\[ \Omega_A = \int P_n d\Omega = \frac{\lambda^2}{A_e} \]

The beam solid angle is the inverse of the effective area, measured in square wavelengths.

- Consider an isotropic antenna, for which:

\[ P_n(\theta, \varphi) \equiv 1 \Rightarrow \int P_n(\theta, \varphi) d\Omega = 4\pi \]

We define **Directive Gain**

a quantity related to resolving power:

\[ D \equiv \frac{4\pi}{\Omega_A} = \frac{4\pi \varepsilon_b}{\Omega_M} = \frac{4\pi \varepsilon_b}{\eta_b \theta_{HP} \varphi_{HP}} \]

Where \( \eta_b \) is a term accounting for the main beam geometry and the subscript ‘HP’ indicates the half-power main beam widths.

**Example:** an effective aperture of diameter 210 m, operating at \( \lambda = 21 \) cm has \( D \sim 10^7 \) or 70 dB.
Radio astronomers often express the effective aperture of a telescope in odd, units, e.g. K/Jy. Here is why. Remember:

then:

\[ A_e [m^2] = \frac{2 \times 1.38 \times 10^{-23}}{10^{-26}} \frac{T_A [K]}{S_{obs} [Jy]} \]

So 1 K/Jy is equivalent to \( \sim 2761 \) m\(^2\), e.g. a 71 m diameter dish with 70% aperture efficiency.

\[ T_A \quad \text{Antenna temperature relates to total detected power p.u. bandwidth} \]

\[ S_{obs} \quad \text{Is the source’s power p.u. bandwidth, } p.u. \text{ of effective area} \]
Again using Nyquist formula, the **System Temperature** is defined as

\[ T_{\text{sys}} \equiv \frac{p_{\text{tot}}}{k} \]

where \( p_{\text{tot}} \) is the **total detected power, including the flux from the source plus everything else:**

\[ T_{\text{sys}} \equiv T_{\text{src}} + T_{\text{sky, bg}} + T_{\text{atmo}} + T_{\text{rx}} + T_{\text{loss}} + T_{\text{spillover}} + T_{\text{rfi}} \]

At 21 cm:

- \( T_{\text{sky, bg}} \): CMB~3K; synchrotron 1-5K f(gal latitude);
- \( T_{\text{atmo}} \): 3K at Zenith
- \( T_{\text{rx}} \): 1-3K
- \( T_{\text{loss}} \): 1-10K
- \( T_{\text{spillover}} \): 5-20K

**\( T_{\text{sys}} \) on “cold sky” (\( T_{\text{src}} \sim 0 \)) \rightarrow 15-40 \text{ K}**

More generally, the term “system temperature” of a receiver+feed+antenna is reserved to the system temperature on “cold sky”.
Radiometer Noise

\[ \sigma_{T,rms} = \frac{kT_{sys}}{\sqrt{t_{int} \times \Delta \nu \times n_{pol}}} \]  
(Where k~1)

Integration time  bandwidth  nr of pols

Continuum Confusion Limit

\[ \sigma_{conf}[\text{Jy}] \approx 3700 \times \nu^{-0.7} \times \Omega_A \]

Discussion:
What is “confusion”?
A way of quantifying the figure of merit of a radio telescope is the **gain**, which is the amount of power – measured by the antenna temperature - received from a source of unit flux density.

It is usually expressed in K/Jy.

It depends on the aperture of the telescope and on the directional characteristics of the feed being used. **The ALFA receiver at Arecibo has a gain G~11 K/Jy in the central feed of the array and ~9.5 K/Jy in the 6 peripheral feeds.**

Gain also depends on telescope configuration w.r.t. the source, i.e. on elevation (zenith angle). In the case of ALFA at Arecibo, gain is fairly constant at ZA<15deg and diminishes progressively at ZA>15deg until the ZA limit of 20deg is reached. This is due to the fact that the “illuminated area” of the primary - i.e. the projection on the surface of the primary of the solid angle from which the feed can receive reflected radiation – spills over into the ground.
**SEFD**

A sometimes used figure of merit of a receiving system is the “System Equivalent Flux Density” or SEFD.

\[
\text{SEFD} = \frac{T_{\text{sys}}}{\text{Gain}}
\]

E.g. a system with \( G = 10 \, \text{K/Jy} \) and \( T_{\text{sys}} = 30 \, \text{K} \) will have a

\[
\text{SEFD} = 3 \, \text{Jy}
\]

And the rms flux density can be expressed in the form

\[
\sigma_{S,rms} = k \times \frac{\text{SEFD}}{\sqrt{t_{\text{int}} \times \Delta \nu \times n_{\text{pol}}}}
\]